

Modified Enlarged 24pt
OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Wednesday 20 May 2020 – Morning

AS Level Mathematics A

H230/02 Pure Mathematics and Mechanics

Time allowed: 1 hour 30 minutes
plus your additional time allowance

YOU MUST HAVE:

the Printed Answer Booklet sent with the standard question paper or any suitable paper provided by the centre. The Printed Answer Booklet may be enlarged by the centre.

a scientific or graphical calculator

YOU CAN USE:

A Model for Question 7

READ INSTRUCTIONS OVERLEAF



INSTRUCTIONS

Use black ink. You can use an HB pencil, but only for graphs and diagrams.

If you use the Printed Answer Booklet write your answer to each question in the space provided. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.

If you use the Printed Answer Booklet fill in the boxes on the front of the Printed Answer Booklet.

Answer ALL the questions.

Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.

Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question.

The acceleration due to gravity is denoted by $g \text{ m s}^{-2}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.

Do NOT send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION

The total number of marks for this paper is 75.

The marks for each question are shown in brackets [].

ADVICE

Read each question carefully before you start your answer.

Formulae

AS Level Mathematics A (H230)

Binomial series

$$(a + b)^n = a^n + {}^nC_1 a^{n-1} b + {}^nC_2 a^{n-2} b^2 + \dots \\ + {}^nC_r a^{n-r} b^r + \dots + b^n \quad (n \in \mathbb{N}),$$

$$\text{where } {}^nC_r = {}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

Differentiation from first principles

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

Standard deviation

$$\sqrt{\frac{\sum (x - \bar{x})^2}{n}} = \sqrt{\frac{\sum x^2}{n} - \bar{x}^2} \quad \text{or}$$

$$\sqrt{\frac{\sum f(x - \bar{x})^2}{\sum f}} = \sqrt{\frac{\sum fx^2}{\sum f} - \bar{x}^2}$$

The binomial distribution

If $X \sim B(n, p)$ then

$P(X = x) = \binom{n}{x} p^x (1 - p)^{n-x}$, mean of X is

np , variance of X is $np(1 - p)$

Kinematics

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$s = \frac{1}{2}(u + v)t$$

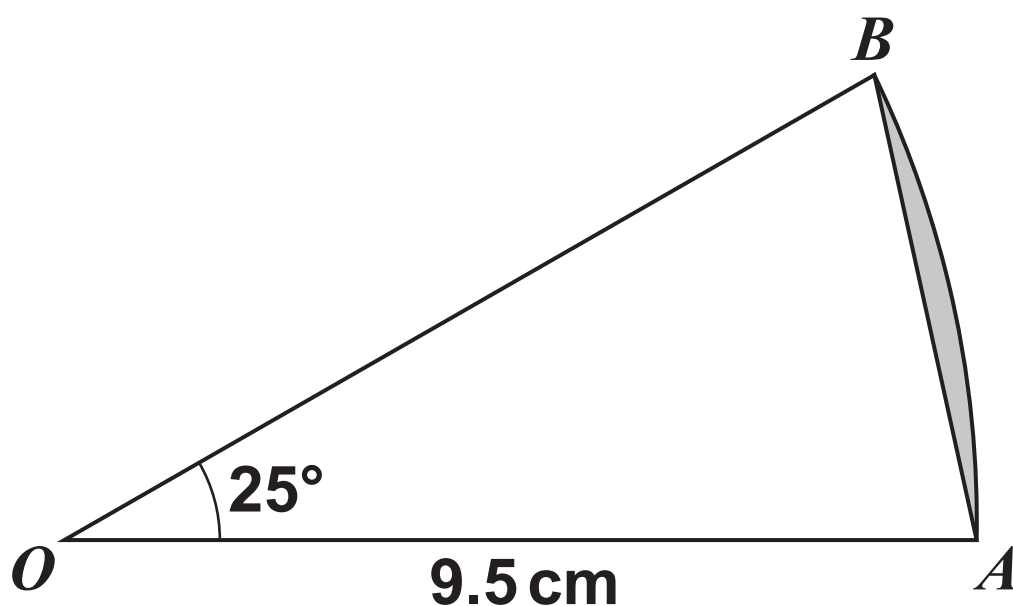
$$v^2 = u^2 + 2as$$

$$s = vt - \frac{1}{2}at^2$$

SECTION A: Pure Mathematics

Answer ALL the questions.

- 1 The diagram below shows a sector AOB of a circle with centre O and radius 9.5 cm . The angle AOB is 25° .



- (a) Calculate the length of the straight line AB . [2]
- (b) Find the area of the segment shaded in the diagram. [3]

2 Two curves have equations $y = \ln x$ and $y = \frac{k}{x}$, where k is a positive constant.

(a) Sketch the curves on a SINGLE diagram. [3]

(b) Explain how your diagram shows that the equation $x \ln x - k = 0$ has exactly one real root. [2]

3 In this question you must show detailed reasoning.

Find the equation of the normal to the curve $y = 4\sqrt{x} - 3x + 1$ at the point on the curve where $x = 4$. Give your answer in the form $ax + by + c = 0$, where a , b and c are integers. [7]

- 4 In this question you must show detailed reasoning.**

The cubic polynomial

$6x^3 + kx^2 + 57x - 20$ is denoted by $f(x)$. It is given that $(2x - 1)$ is a factor of $f(x)$.

(a) Use the factor theorem to show that $k = -37$. [2]

(b) Using this value of k , factorise $f(x)$ completely. [3]

(c) (i) Hence find the three values of t satisfying the equation

$$6e^{-3t} - 37e^{-2t} + 57e^{-t} - 20 = 0. \text{ [2]}$$

(ii) Express the sum of the three values found in part (c)(i) as a single logarithm. [2]

- 5 A curve has equation $y = a(x + b)^2 + c$, where a , b and c are constants. The curve has a stationary point at $(-3, 2)$.**

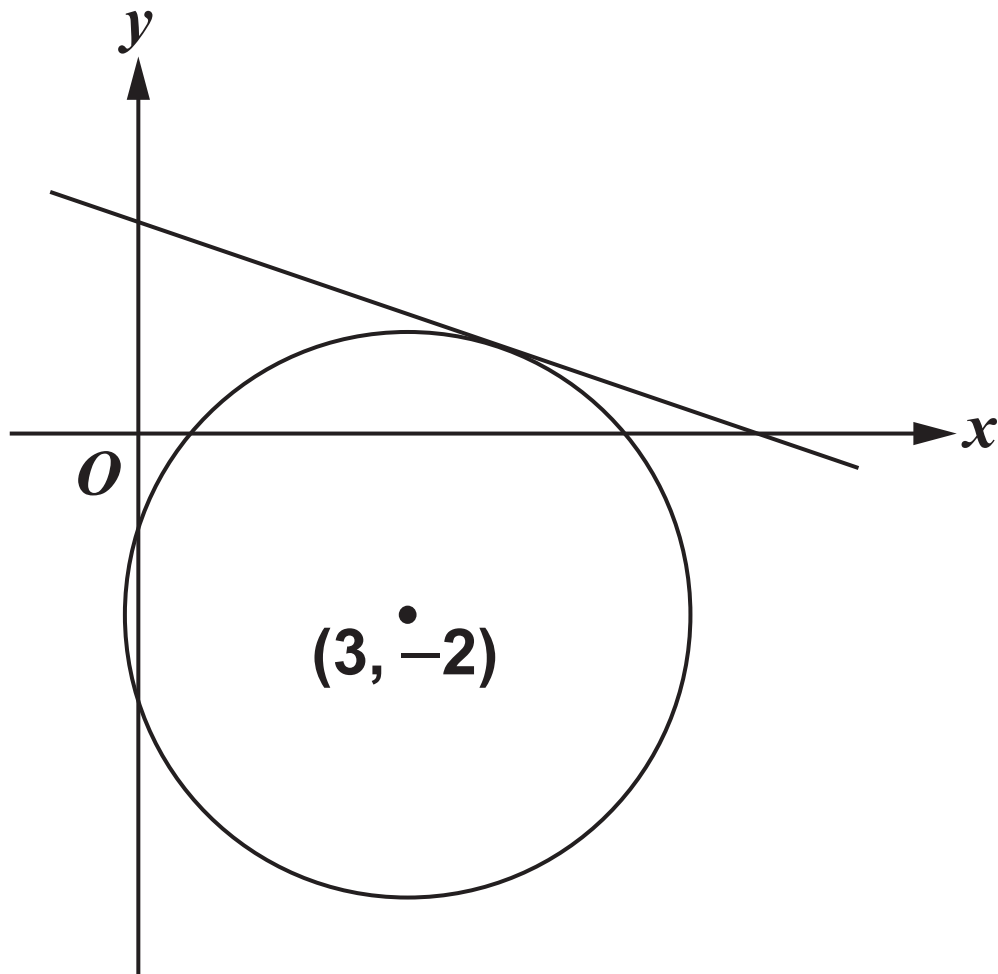
(a) State the values of b and c . [2]

When the curve is translated by $\begin{pmatrix} 4 \\ 0 \end{pmatrix}$ the transformed curve passes through the point $(3, -18)$.

(b) Determine the value of a . [3]

6 In this question you must show detailed reasoning.

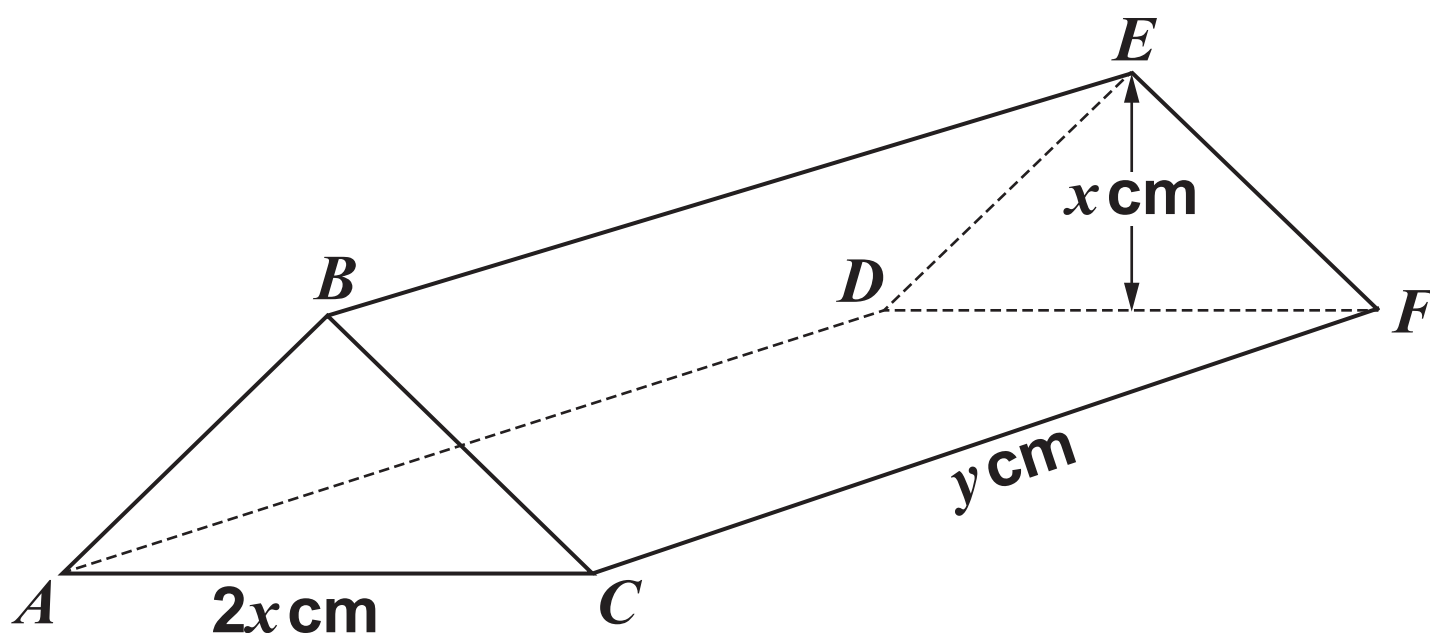
The diagram below shows the line $3y + x = 7$ which is a tangent to a circle with centre $(3, -2)$.



Find an equation for the circle. [6]

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- 7 The diagram below shows a model for the roof of a toy building. You may also use a model to help you. The roof is in the form of a solid triangular prism $ABCDEF$. The base $ACFD$ of the roof is a horizontal rectangle, and the cross-section ABC of the roof is an isosceles triangle with $AB = BC$.



The lengths of AC and CF are $2x \text{ cm}$ and $y \text{ cm}$ respectively, and the height of BE above the base of the roof is $x \text{ cm}$.

The total surface area of the FIVE faces of the roof is 600 cm^2 and the volume of the roof is $V \text{ cm}^3$.

- (a) Show that $V = kx(300 - x^2)$, where $k = \sqrt{a} + b$ and a and b are integers to be determined. [6]
- (b) Use differentiation to determine the value of x for which the volume of the roof is a maximum. [4]
- (c) Find the maximum volume of the roof. Give your answer in cm^3 , correct to the nearest integer. [1]
- (d) Explain why, for this roof, x must be less than a certain value, which you should state. [2]

SECTION B: Mechanics

Answer ALL the questions.

- 8 A particle is in equilibrium under the action of the following three forces:**

$(2p\mathbf{i} - 4\mathbf{j})$ N, $(-3q\mathbf{i} + 5p\mathbf{j})$ N and $(-13\mathbf{i} - 6\mathbf{j})$ N.

Find the values of p and q . [3]

- 9 A crane lifts a car vertically. The car is inside a crate which is raised by the crane by means of a strong cable. The cable can withstand a maximum tension of 9500 N without breaking. The crate has a mass of 55 kg and the car has a mass of 830 kg.**

(a) Find the maximum acceleration with which the crate and car can be raised. [2]

- (b) Show on a clearly labelled diagram the forces acting on the CRATE while it is in motion. [1]
- (c) Determine the magnitude of the reaction force between the crate and the car when they are ascending with maximum acceleration. [3]

10 A particle P is moving in a straight line. At time t seconds P has velocity $v \text{ m s}^{-1}$ where $v = (2t + 1)(3 - t)$.

- (a) Find the deceleration of P when $t = 4$. [2]
- (b) State the positive value of t for which P is instantaneously at rest. [1]
- (c) Find the total distance that P travels between times $t = 0$ and $t = 4$. [3]

11 A car starts from rest at a set of traffic lights and moves along a straight road with constant acceleration 4 m s^{-2} . A motorcycle, travelling parallel to the car with constant speed 16 m s^{-1} , passes the same traffic lights exactly 1.5 seconds after the car starts to move. The time after the car starts to move is denoted by t seconds.

(a) Determine the two values of t at which the car and motorcycle are the same distance from the traffic lights. [6]

These two values of t are denoted by t_1 and t_2 , where $t_1 < t_2$.

(b) Describe the relative positions of the car and the motorcycle when $t_1 < t < t_2$. [1]

(c) Determine the maximum distance between the car and the motorcycle when $t_1 < t < t_2$. [3]

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